Mechanics of Materials (7th Edition)

Chapter 7, Problem 2CP

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Problem

A student bumbarked the following problem:

A state of plane stress is defined by the stress components $(\sigma_x)_1, (\sigma_y)_1$ and $(\tau_{xy})_1$ associated with the element shown in Fig. P7.2/a. (a) Write a computer program that can be used to calculate the principal stress, the principal directions, the maximum in-plane shearing stress, and the maximum shearing stress. (b) Use this program to solve Probs. 7.5, 7.9, 7.65, and 7.69.

Reference 1

For the given state of stress, determine (a) the principal planes, (b) the principal stresses

Reference 2

For the given state of stress, determine (a) the orientation of the planes of maximum in-plane shearing stress, (b) the maximum in-plane shearing stress, (c) the corresponding normal stress

Reference 3

For the state of stress shown, determine the maximum shearing stress when

$(a)\ \sigma_x = 40\ \text{MPa}, (b)\ \sigma_x = 120\ \text{MPa},$ HINT Consider both in-plane and out-of-plane shearing stresses.

Reference 4

For the state of stress shown, determine the maximum shearing stress when

$(a)\ \sigma_x = 20\ \text{MPa}, (b)\ \sigma_x = 140\ \text{MPa}.$ HINT Consider both in-plane and out-of-plane shearing stresses.

Step-by-step solution

Step 1 of 15

For an element under plane stress condition, the location of the principal axes $\left[\theta_p\right]$ and $\left[\theta_s\right]$, the principal stresses $\left[\sigma_{max}\right]$ and $\left[\sigma_{min}\right]$, the maximum in-plane shearing stress $\left[\tau_{max(in-plane)}\right]$, and the maximum shearing stress $\left[\tau_{max(absolute)}\right]$ are calculated at the surfaces of the element (shown in the figure below) as,

For the given stress element,

Comment

Step 2 of 15

To compute principal stress on the given stress element, average normal stress $\left[\sigma_{avg}\right]$ and radius $\left[R\right]$ of the Mohr circle are to be evaluated first, as,

Average normal stress $\left[\sigma_{avg}\right]$ is calculated as,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad \ldots (1)$$

Radius $\left[R\right]$ of the Mohr circle is calculated as,

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \ldots (2)$$

Comment

Step 3 of 15

Now, principal stresses $\left[\sigma_{max}\right]$ and $\left[\sigma_{min}\right]$ that act on the stress element are evaluated (from equation 1 and 2) as,

Maximum principal stress $\left[\sigma_{max}\right]$ is calculated as,

$$\sigma_{max} = \sigma_{avg} + R \quad \ldots (3)$$

Minimum principal stress $\left[\sigma_{min}\right]$ is calculated as,

$$\sigma_{min} = \sigma_{avg} - R \quad \ldots (4)$$

Comment

Step 4 of 15

Principal planes are located the angle $\left[\theta_p\right]$ and $\left[\theta_s\right]$ as,

Angle between $\left[\right]$ axes and principal axes that is $\left[\theta_p\right]$ is calculated as,

$$2\theta_p = \tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Angle between $\left[\right]$ axes and planes of maximum in-plane shearing stress (that is, $\left[\theta_s\right]$) is calculated as,

$$2\theta_s = \tan^{-1}\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Comment

Step 5 of 15

Shearing stress conditions are evaluated to compute the maximum in-plane shearing stress $\left[\tau_{max(in-plane)}\right]$ and the maximum shearing stress $\left[\tau_{max(absolute)}\right]$ as,

Case 1:

If, $\sigma_{avg} > 0$ and $\sigma_{min} < 0$.

Then,

$\tau_{max(in-plane)} = R$, and

$\tau_{max(absolute)} = R$

Case 2:

If, $\sigma_{avg} > 0$ and $\sigma_{min} > 0$.

Then,

$\tau_{max(in-plane)} = R$, and

$\tau_{max(absolute)} = \frac{\sigma_{max}}{2}$

Case 3:

If, $\sigma_{avg} < 0$ and $\sigma_{max} < 0$.

Then,

$\tau_{max(in-plane)} = R$, and

$\tau_{max(absolute)} = \left|\frac{\sigma_{min}}{2}\right|$

Comment

Step 6 of 15

(a)

Type the following code in the **MATLAB** to obtain the solution for the provided questions.

```
function [theta_p1,max_in_sigma,max_sigma,min_theta_s1,theta_s2,...
    min_in_sigma,min_sigma,max_theta_s1]=principal_in_sigma_2_1(sx,...
    % Initialize
    sigma_avg = (sigma_x + sigma_y)/2;
    % Calculate Radius of the Mohr circle
    R = sqrt(((sigma_x) - (sigma_y))^2 + (2*sy^2));
    % Calculate maximum principal stress
    sigma_max = sigma_avg + R;
    % Calculate minimum principal stress
    sigma_min = sigma_avg - R;
    % Locate the angles of the principal planes (perpendicular to each other)
    theta_p1 = (atan2((2*(sy)),((sigma_x) - (sigma_y))))/2;
    theta_p2 = theta_p1 + 90;
    % Locate the position of the maximum shear stress
    theta_s1 = (atan2(-((sigma_x) - (sigma_y)),(2*(sy))))/2;
    theta_s2 = theta_s1 + 90;
    % Calculate maximum shear stress
    if(sigma_avg > 0 & sigma_min < 0)
        T_max_in = R;
        T_max = R;
    elseif(sigma_avg > 0 & sigma_min > 0)
        T_max_in = R;
        T_max = (max(sigma_max));
    elseif(sigma_avg < 0 & sigma_max < 0)
        T_max_in = R;
        T_max = abs(min(sigma_min));
    end
    % Print the output values
    fprintf('\nAngle between sx axes and principal axes (+ Counter-clockwise): %
    fprintf('\nPrincipal plane is at %0.2f Degrees, and %0.2f Degrees',...
        theta_p1,theta_p2);
    fprintf('\nSigma_max = %0.2f MPa\nSigma_min = %0.2f MPa\n',sigma_max,sigma_min)
    fprintf('\n')
    fprintf('\nAngle between sx axes and planes of maximum in-plane shearing :...
        theta_s1);
    fprintf('\n')
    fprintf('\nPlane of maximum in-plane shearing is at %0.2f Degrees, and :...
        %0.2f Degrees\n',theta_s1,theta_s2);
    fprintf('\nMaximum in-plane shearing stress(Tau_max)in MPa\n\n',...
        T_max_in,T_max)
end
```

Comment

Step 7 of 15

Solution for Problem 7.5 and 7.9.

For given input parameters,

$\sigma_x = 40$ MPa,

$\sigma_y = 40$ MPa, and

$\tau_{xy} = 0$ MPa.

Comment

Step 8 of 15

Execute the MATLAB code, to obtain the output for the given problem as,

Type following input in the command window:

```
>> [theta_p1,theta_p2,sigma_max,sigma_min,theta_s1,theta_s2,...
        T_max_in,T_max]=P_7_2_CH_40_40_0()
```

The obtained result for the provided values is,

```
Angle between sy axes and principal axes (+ Counter-clockwise):

Principal plane is at -37.83 Degrees, and 52.17 Degrees
Sigma_max = 75.00 MPa
Sigma_min = -40.00 MPa


Angle between sy axes and planes of maximum in-plane shearing stress :

Plane of maximum shear stress is at -7.83 Degrees, and 97.87 Degrees
        T_max(in-plane) = 35.00 MPa
T_max(out-of-plane) = 40.00 MPa
```

Comment

Step 9 of 15

Solution for Problem 7.65.

For given input parameters,

$\sigma_x = 140$ MPa,

$\sigma_y = 40$ MPa, and

$\tau_{xy} = 60$ MPa.

Comment

Step 10 of 15

Execute the MATLAB code, to obtain the output for the given problem as,

Type following input in the command window:

```
>> [theta_p1,theta_p2,sigma_max,sigma_min,theta_s1,theta_s2,...
        T_max_in,T_max]= P_7_2_CH_140_40_60()
```

The obtained result for the provided values is,

```
Angle between sy axes and principal axes (+ Counter-clockwise):

Principal plane is at 25.00 Degrees, and 115.00 Degrees
sigma_max = 150.00 MPa
sigma_min = -10.00 MPa


Angle between sy axes and planes of maximum in-plane shearing stress :

Plane of maximum shear stress is at -20.00 Degrees, and 70.00 Degrees
        T_max(in-plane) = 80.00 MPa
T_max(out-of-plane) = 80.00 MPa
```

Comment

Step 11 of 15

For given input parameters,

$\sigma_x = 140$ MPa,

$\sigma_y = 120$ MPa, and

$\tau_{xy} = 60$ MPa.

Comment

Step 12 of 15

Execute the MATLAB code, to obtain the output for the given problem as,

Type following input in the command window:

```
>> [theta_p1,theta_p2,sigma_max,sigma_min,theta_s1,theta_s2,...
        T_max_in,T_max]= P_7_2_CH_140_120_60()
```

The obtained result for the provided values is,

```
Angle between sy axes and principal axes (+ Counter-clockwise):

Principal plane is at 41.84 Degrees, and 131.84 Degrees
sigma_max = 210.12 MPa
sigma_min = 49.36 MPa


Angle between sy axes and planes of maximum in-plane shearing stress :

Plane of maximum shear stress is at -3.16 Degrees, and 86.84 Degrees
        T_max(in-plane) = 80.12 MPa
T_max(out-of-plane) = 105.31 MPa
```

Comment

Step 13 of 15

Solution for Problem 7.69.

For given input parameters,

$\sigma_x = 140$ MPa,

$\sigma_y = 20$ MPa, and

$\tau_{xy} = 60$ MPa.

Execute the MATLAB code, to obtain the output for the given problem as,

Type following input in the command window:

```
>> [theta_p1,theta_p2,sigma_max,sigma_min,theta_s1,theta_s2,...
        T_max_in,T_max]= P_7_2_CH_140_20_60()
```

The obtained result for the provided values is,

```
Angle between sy axes and principal axes (+ Counter-clockwise):

Principal plane is at 27.57 Degrees, and 116.57 Degrees
sigma_max = 160.00 MPa
sigma_min = -20.00 MPa


Angle between sy axes and planes of maximum in-plane shearing stress :

Plane of maximum shear stress is at -18.43 Degrees, and 71.57 Degrees
        T_max(in-plane) = 90.00 MPa
T_max(out-of-plane) = 100.00 MPa
```

Comment

Step 14 of 15

For given input parameters,

$\sigma_x = 140$ MPa,

$\sigma_y = 140$ MPa, and

$\tau_{xy} = 60$ MPa.

Comment

Step 15 of 15

Execute the MATLAB code, to obtain the output for the given problem as,

Type following input in the command window:

```
>> [theta_p1,theta_p2,sigma_max,sigma_min,theta_s1,theta_s2,...
        T_max_in,T_max]= P_7_2_CH_140_140_60()
```

The obtained result for the provided values is,

```
Angle between sy axes and principal axes (+ Counter-clockwise):

Principal plane is at 45.00 Degrees, and 135.00 Degrees
sigma_max = 220.00 MPa
sigma_min = 60.00 MPa


Angle between sy axes and planes of maximum in-plane shearing stress :

Plane of maximum shear stress is at -0.00 Degrees, and 90.00 Degrees
        T_max(in-plane) = 80.00 MPa
T_max(out-of-plane) = 110.00 MPa
```

Comment

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